Third Canadian Edition



JOSEPH F. HEALEY

STEVEN G. PRUS

FREQUENTLY USED FORMULAS

CHAPTER 2

Proportion

$$p = \frac{f}{n}$$

Percentage

$$\% = \left(\frac{f}{n}\right) \times 100$$

CHAPTER 3

Mean (sample)

$$\overline{X} = \frac{\Sigma(X_i)}{n}$$

Standard deviation (sample)

$$s = \sqrt{\frac{\Sigma(X_i - \overline{X})^2}{n}}$$

CHAPTER 4

Zscores

$$Z = \frac{X_i - \overline{X}}{s}$$

CHAPTER 6

Confidence interval for a sample mean

c.i. =
$$\overline{X} \pm Z\left(\frac{s}{\sqrt{n-1}}\right)$$

Confidence interval for a sample proportion

$$\text{c.i.} = P_s \pm Z \sqrt{\frac{P_u(1-P_u)}{n}}$$

CHAPTER 7

Means

$$t$$
(obtained) = $\frac{\overline{X} - \mu}{s/\sqrt{n-1}}$

Proportions

$$Z(\text{obtained}) = \frac{P_s - P_u}{\sqrt{P_u(1 - P_u)/n}}$$

CHAPTER 8

$$Z(\text{obtained}) = \frac{(\overline{X}_1 - \overline{X}_2)}{\sigma_{\overline{x} - \overline{x}}}$$

Standard deviation of the sampling distribution for sample means

$$\sigma_{\bar{x}-\bar{x}} = \sqrt{\frac{s_1^2}{n_1 - 1} + \frac{s_2^2}{n_2 - 1}}$$

Pooled estimate of population proportion

$$P_{u} = \frac{n_{1}P_{s1} + n_{2}P_{s2}}{n_{1} + n_{2}}$$

Standard deviation of the sampling distribution for sample proportions

$$\sigma_{p-p} = \sqrt{P_u(1-P_u)} \sqrt{(n_1 + n_2)/n_1 n_2}$$

Proportions

$$Z(\text{obtained}) = \frac{(P_{s1} - P_{s2})}{\sigma_{p-p}}$$

CHAPTER 9

Total sum of squares

$$SST = \sum (X_i - \overline{X})^2$$

Sum of squares between

$$SSB = \sum n_k (\overline{X}_k - \overline{X})^2$$

Sum of squares within

$$SSW = \sum (X_i - X_k)^2$$

Degrees of freedom for SSW dfw = n - k

Degrees of freedom for SSB

$$dfb = k - 1$$

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.

Mean square within

$$MSW = \frac{SSW}{dfw}$$

Mean square between

$$MSB = \frac{SSB}{dfb}$$

F ratio

 $F = \frac{\text{MSB}}{\text{MSW}}$

CHAPTER 10

Chi square

$$\chi^2$$
 (obtained) = $\Sigma \frac{(f_o - f_e)^2}{f_e}$

CHAPTER 11

Phi

$$\phi = \sqrt{\frac{\chi^2}{n}}$$

Cramer's V

$$V = \sqrt{\frac{\chi^2}{(n)(\text{Minimum of } r-1, c-1)}}$$

Lambda

$$\lambda = \frac{E_1 - E_2}{E_1}$$

CHAPTER 12

Gamma

$$G = \frac{n_s - n_d}{n_s + n_d}$$

Spearman's rho

$$r_{s} = 1 - \frac{6\Sigma D^{2}}{n(n^{2} - 1)}$$

CHAPTER 13

Least-squares regression line

$$Y = a + bX$$

Slope

$$b = \frac{n \Sigma XY - (\Sigma X)(\Sigma Y)}{n \Sigma X^2 - (\Sigma X)^2}$$

$$a = \overline{Y} - b\overline{X}$$

Pearson's r

$$r = \frac{n\Sigma XY - (\Sigma X)(\Sigma Y)}{\sqrt{[n\Sigma X^2 - (\Sigma X)^2][n\Sigma Y^2 - (\Sigma Y)^2]}}$$

CHAPTER 14

Partial correlation coefficient

$$r_{yx,z} = \frac{r_{yx} - (r_{yz})(r_{xz})}{\sqrt{1 - r_{yz}^2}\sqrt{1 - r_{xz}^2}}$$

Least-squares multiple regression line

$$Y = a + b_1 X_1 + b_2 X_2$$

Partial slope for X_1

$$b_1 = \left(\frac{s_y}{s_1}\right) \left(\frac{r_{y1} - r_{y2}r_{12}}{1 - r_{12}^2}\right)$$

Partial slope for X_2

$$b_2 = \left(\frac{s_y}{s_2}\right) \left(\frac{r_{y2} - r_{y1}r_{12}}{1 - r_{12}^2}\right)$$

Y intercept

$$a = \overline{Y} - b_1 \overline{X}_1 - b_2 \overline{X}_2$$

Beta-weight for X_1

$$b_1^* = b_1 \left(\frac{s_1}{s_y}\right)$$

Beta-weight for X_2

$$b_2^* = b_2 \left(\frac{s_2}{s_y}\right)$$

Standardized least-squares regression line

$$Z_{y} = b_{1}^{*}Z_{1} + b_{2}^{*}Z_{2}$$

Coefficient of multiple determination

$$R^{2} = r_{y1}^{2} + r_{y2.1}^{2}(1 - r_{y1}^{2})$$

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.



Third Canadian Edition

Joseph F. Healey Christopher Newport University

Steven G. Prus

Carleton University



Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.

This is an electronic version of the print textbook. Due to electronic rights restrictions, some third party content may be suppressed. The publisher reserves the right to remove content from this title at any time if subsequent rights restrictions require it. For valuable information on pricing, previous editions, changes to current editions, and alternate formats, please visit nelson.com to search by ISBN#, author, title, or keyword for materials in your areas of interest.



Statistics: A Tool for Social Research, Third Canadian Edition by Joseph F. Healey and Steven G. Prus

Vice President, Editorial Higher Education: Anne Williams

Publisher: Leanna MacLean

Marketing Manager: Terry Fedorkiw

Technical Reviewer: Chris Esselmont

Developmental Editor: Jessica Freedman

Photo Researcher Melody Tolson

COPYRIGHT © 2016, 2013 by Nelson Education Ltd.

Adapted from *Statistics: A Tool* for *Social Research*, Tenth Edition, by Joseph F. Healey, published by Cengage Learning. Copyright © 2015 by Cengage Learning.

Printed and bound in the United States 2 3 4 5 19 18 17 16

For more information contact Nelson Education Ltd., 1120 Birchmount Road, Toronto, Ontario, M1K 5G4. Or you can visit our Internet site at http://www.nelson.com Permissions Coordinator: Melody Tolson

Production Service: Integra Software Services Pvt. Ltd.

Copy Editor: Matthew Kudelka

Proofreader: Integra Software Services Pvt. Ltd.

Indexer: Integra Software Services Pvt. Ltd.

Design Director: Ken Phipps

ALL RIGHTS RESERVED. No part of this work covered by the copyright herein may be reproduced, transcribed, or used in any form or by any means—graphic, electronic, or mechanical, including photocopying, recording, taping, Web distribution, or information storage and retrieval systems without the written permission of the publisher.

For permission to use material from this text or product, submit all requests online at www.cengage.com/permissions. Further questions about permissions can be emailed to permissionrequest@cengage.com

Every effort has been made to trace ownership of all copyrighted material and to secure permission from copyright holders. In the event of any question arising as to the use of any material, we will be pleased to make the necessary corrections in future printings. Managing Designer: Franca Amore

Interior Design Modifications: Peter Papayanakis

Cover Design: Courtney Hellam

Cover Image: grmarc/Shutterstock

Compositor: Integra Software Services Pvt. Ltd.

Library and Archives Canada Cataloguing in Publication Data

Healey, Joseph F., 1945–, author Statistics : a tool for social research / Joseph F. Healey (Christopher Newport University), Steven G. Prus (Carleton University). — Third Canadian edition. Includes bibliographical references and index. ISBN 978-0-17-656207-6 (pbk.)

1. Social sciences—Statistical methods. 2. Statistics. I. Prus, Steven G. (Steven Gerald), 1967-, author II. Title.

HA29.H42 2015 519.5 C2014-907520-0

ISBN-13: 978-0-17-656207-6 ISBN-10: 0-17-656207-9

Brief Contents

Preface		xiv
Prologue: Basic Mathematics Review		1
Chapter 1	Introduction 10	
Part 1	Descriptive Statistics	36
Chapter 2	Basic Descriptive Statistics: Percentages, Ratios and Rates, Tables, Charts, and Graphs 38	
Chapter 3 Chapter 4	Measures of Central Tendency and Dispersion 76 The Normal Curve 113	
Part 2	Inferential Statistics	142
Chapter 5	Introduction to Inferential Statistics: Sampling and the Sampling Distribution 144	
Chapter 6	Estimation Procedures 162	
Chapter 7	Hypothesis Testing I: The One-Sample Case 195	
Chapter 8	Hypothesis Testing II: The Two-Sample Case 235	
Chapter 9	Hypothesis Testing III: The Analysis of Variance 268	
Chapter 10	Hypothesis Testing IV: Chi Square 298	
Part 3	Bivariate Measures of Association	330
Chapter 11	Introduction to Bivariate Association and Measures of Association for Variables Measured at the Nominal Level 332	
Chapter 12	Association Between Variables Measured at the Ordinal Level 363	
Chapter 13	Association Between Variables Measured at the Interval-Ratio Level 404	
Part 4	Multivariate Techniques	442
Chapter 14	Partial Correlation and Multiple Regression and Correlation	444
Appendix A	Area Under the Normal Curve	481
Appendix B	Distribution of t	485
Appendix C	Distribution of Chi Square	486
Appendix D Distribution of F		

Appendix E Using Statistics: Ideas for Research Projects	489
Appendix F An Introduction to IBM SPSS Statistics for Windows	495
Appendix G Information and Code Books for the 2012 Canadian Community Health Survey and 2009 General Social Survey	510
Answers to Odd-Numbered Computational Problems	527
Glossary	539
Index	545

Detailed Contents

Pref	ace	Х
Prol	ogue: Basic Mathematics Review	
Cha	pter 1 Introduction	
1.1	Why Study Statistics? 10	
1.2	The Role of Statistics in Scientific Inquiry 11	
1.3	The Goals of This Textbook 15	
1.4	Descriptive and Inferential Statistics 16	
1.5	Discrete and Continuous Variables 19	
1.6	Level of Measurement 21	
	Reading Statistics 1: Introduction 24	
	Summary 29 / Glossary 30 / Problems 30	
	You Are the Researcher: Introduction to the Canadian General	
	Social Survey, Canadian Community Health Survey, and SPSS 33	
Par	t 1 DESCRIPTIVE STATISTICS	-
Cha	pter 2 Basic Descriptive Statistics: Percentages, Ratios and Rates,	
Cha Tab	pter 2 Basic Descriptive Statistics: Percentages, Ratios and Rates, les, Charts, and Graphs	
Cha	pter 2 Basic Descriptive Statistics: Percentages, Ratios and Rates, les, Charts, and Graphs Percentages and Proportions 39	
Cha Tab	 Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 	
Cha Tab 2.1	Applying Statistics 2.2: Ratios 42	
Cha Tab	 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 	
Cha Tab 2.1 2.2	 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 	
Cha Tab 2.1 2.2 2.3	 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 	
Cha Tab 2.1 2.2	 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 	
Cha Tab 2.1 2.2 2.3 2.4	 Applying Statistics 2.3: Rates 44 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 	
Cha Tab 2.1 2.2 2.3	 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal 	
Cha Tab 2.1 2.2 2.3 2.4	 Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 Frequency Distributions for Variables Measured at the Interval-Ratio Level 49 	
Cha Tab 2.1 2.2 2.3 2.4 2.5	 Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 Frequency Distributions for Variables Measured at the Interval-Ratio 	
Cha Tab 2.1 2.2 2.3 2.4 2.5	 apter 2 Basic Descriptive Statistics: Percentages, Ratios and Rates, les, Charts, and Graphs Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 Frequency Distributions for Variables Measured at the Interval-Ratio Level 49 Charts and Graphs 59 	
Cha Tab 2.1 2.2 2.3 2.4 2.5	 Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 Frequency Distributions for Variables Measured at the Interval-Ratio Level 49 Charts and Graphs 59 Reading Statistics 2: Percentages, Rates, Tables, and Graphs 64 	
Cha Tab 2.1 2.2 2.3 2.4 2.5	 Percentages and Proportions 39 Applying Statistics 2.1: Communicating with Statistics 41 Applying Statistics 2.2: Ratios 42 Ratios and Rates 43 Applying Statistics 2.3: Rates 44 Frequency Distributions Introduction 46 Frequency Distributions for Variables Measured at the Nominal and Ordinal Levels 47 Frequency Distributions for Variables Measured at the Interval-Ratio Level 49 Charts and Graphs 59 Reading Statistics 2: Percentages, Rates, Tables, and Graphs 64 Summary 65 / Summary of Formulas 66 / Glossary 66 / 	

Chapter 3 Measures of Central Tendency and Dispersion

76

- 3.1 Introduction 77
- 3.2 Nominal-Level Measures 79

- 3.3 Ordinal-Level Measures 82
- 3.4 Interval-Ratio-Level Measures 86
 Applying Statistics 3.1: The Mean and Standard Deviation 93
 Applying Statistics 3.2: Describing Dispersion 94
- 3.5 Choosing a Measure of Central Tendency and Dispersion 97
- 3.6 Interpreting Statistics: The Central Tendency and Dispersion of Income in Canada 98 *Reading Statistics 3: Measures of Central Tendency and Dispersion* 101
 Summary 102 / Summary of Formulas 103 / Glossary 103 / Multimedia Resources 104 / Problems 104
 You Are the Researcher: Using SPSS to Compute Measures of Central Tendency and Dispersion with the 2012 CCHS 109

Chapter 4 The Normal Curve

113

138

142

144

- 4.1 Introduction 114
- 4.2 Computing Z Scores 117
- 4.3 The Normal Curve Table 119
- 4.4 Finding Total Area Above and Below a Score 121
- 4.5 Finding Areas Between Two Scores 123 Applying Statistics 4.1: Finding the Area Below or Above a Z Score 126
- 4.6 Using the Normal Curve to Estimate Probabilities 126 *Applying Statistics 4.2: Finding the Area Between Z Scores* 127 *Applying Statistics 4.3: Finding Probabilities* 128 Summary 131 / Summary of Formulas 131 / Glossary 132 / Multimedia Resources 132 / Problems 132 You Are the Researcher: Using SPSS to Produce Histograms and Compute *Z* Scores with the 2012 CCHS 134

Part 1 Cumulative Exercises

Part 2 INFERENTIAL STATISTICS

Chapter 5 Introduction to Inferential Statistics: Sampling and the Sampling Distribution

- 5.1 Introduction 144
- 5.2 Probability Sampling 145
- 5.3 The Sampling Distribution 147
- 5.4 Constructing the Sampling Distribution 151
- 5.5 Linking the Population, Sampling Distribution, and Sample 155
- 5.6 Symbols and Terminology 157
 Summary 158 / Glossary 158 / Multimedia Resources 159 / Problems 159
 You Are the Researcher: Using SPSS to Draw Random Samples with the 2012 CCHS 160

162

Chapter 6 Estimation Procedures

- 6.1 Introduction 162
- 6.2 Bias and Efficiency 163
- 6.3 Estimation Procedures: Introduction 166
- 6.4 Interval Estimation Procedures for Sample Means (Large Samples) 169 Applying Statistics 6.1: Estimating a Population Mean 170
- 6.5 Graphing a Confidence Interval of a Sample Mean 172
- 6.6 Interval Estimation Procedures for Sample Proportions (Large Samples) 173 Applying Statistics 6.2: Estimating Population Proportions 174
- 6.7 A Summary of the Computation of Confidence Intervals 175
- 6.8 Controlling the Width of Interval Estimates 176
- 6.9 Determining Sample Size 178
- 6.10 Interpreting Statistics: Predicting the Election of the Government of Canada and Judging Its Performance 180 *Reading Statistics 4: Polls* 184 *Reading Statistics 5: Using Representative Samples to Track National Trends* 186 Summary 187 / Summary of Formulas 188 / Glossary 188 / Multimedia Resources 188 / Problems 189 You Are the Researcher: Using SPSS to Produce Confidence Intervals with the 2012 CCHS 191

Chapter 7 Hypothesis Testing I: The One-Sample Case

195

- 7.1 Introduction 196
- 7.2 An Overview of Hypothesis Testing 197
- 7.3 The Five-Step Model for Hypothesis Testing 203 Applying Statistics 7.1: Testing a Sample Mean for Significance 207
- 7.4 One-Tailed and Two-Tailed Tests of Hypothesis 208
- 7.5 Selecting an Alpha Level 214
- 7.6 The Student's t Distribution 216
- 7.7 Tests of Hypotheses for Single-Sample Proportions (Large Samples) 221 Applying Statistics 7.2: Testing a Sample Proportion for Significance 223
- 7.8 Hypothesis Testing Using Confidence Intervals 225
 Summary 227 / Summary of Formulas 227 / Glossary 228 / Multimedia Resources 228 / Problems 229
 You Are the Researcher: Using SPSS to Conduct a One-Sample Test with the 2012 CCHS 232

Chapter 8 Hypothesis Testing II: The Two-Sample Case

235

- 8.1 Introduction 236
- 8.2 Hypothesis Testing with Sample Means (Large Samples) 236 Applying Statistics 8.1: Testing the Significance of the Difference Between Sample Means 241
- 8.3 Hypothesis Testing with Sample Means (Small Samples) 242

- 8.4 Hypothesis Testing with Sample Proportions (Large Samples) 245
- 8.5 Hypothesis Testing Using Individual Confidence Intervals 248
- 8.6 The Limitations of Hypothesis Testing: Significance vs. Importance 250 Applying Statistics 8.2: Testing the Significance of the Difference Between Sample Proportions 251
- 8.7 Interpreting Statistics: Are There Significant Differences in Income Between Men and Women? 253 *Reading Statistics 6: Hypothesis Testing* 255 Summary 256 / Summary of Formulas 257 / Glossary 257 / Multimedia Resources 258 / Problems 258 You Are the Researcher: Using SPSS to Test the Significance of the Difference Between Two Means with the 2009 GSS 262

Chapter 9 Hypothesis Testing III: The Analysis of Variance

268

- 9.1 Introduction 268
- 9.2 The Logic of the Analysis of Variance 269
- 9.3 The Computation of ANOVA 271
- 9.4 A Computational Shortcut 273
- 9.5 A Computational Example 274
- 9.6 A Test of Significance for ANOVA 276 Applying Statistics 9.1: The Analysis of Variance 278
- 9.7 An Additional Example for Computing and Testing the Analysis of Variance 280
- 9.8 The Limitations of the Test 282
- 9.9 Interpreting Statistics: Does Personal Well-Being Vary by Marital Status 283 Summary 287 / Summary of Formulas 287 / Glossary 288 / Multimedia Resources 288 / Problems 288 You Are the Researcher: Using SPSS to Conduct Analysis of Variance with the 2012 CCHS 292

Chapter 10 Hypothesis Testing IV: Chi Square

298

- 10.1 Introduction 298
- 10.2 Bivariate Tables 299
- 10.3 The Logic of Chi Square 300
- 10.4 The Computation of Chi Square 302
- 10.5 The Chi Square Test for Independence 305 Applying Statistics 10.1: The Chi Square Test 309
- 10.6 The Chi Square Test: An Example 310
- 10.7 The Limitations of the Chi Square Test 312
 Applying Statistics 10.2: The Chi Square Test in a Larger (2 × 4) Table 312
 Reading Statistics 7: Gender Differences in Health 313

	Summary 316 / Summary of Formulas 316 / Glossary 317 / Multime Resources 317 / Problems 317 You Are the Researcher: Using SPSS to Conduct the Chi Square Test with the 2009 GSS 321	dia
Part	2 Cumulative Exercises	326
Part	t 3 BIVARIATE MEASURES OF ASSOCIATION	330
	pter 11 Introduction to Bivariate Association and Measures ssociation for Variables Measured at the Nominal Level	332
11.2 11.3 11.4 11.5 11.6 11.7	Statistical Significance and Theoretical Importance 333 Association Between Variables and the Bivariate Table 334 Three Characteristics of Bivariate Associations 336 <i>Applying Statistics 11.1: Association and Bivariate Tables 338</i> The Importance of Percentages and Some Errors of Interpretation 342 Introduction to Measures of Association 343 Chi Square–Based Measures of Association 344 Lambda: A Proportional Reduction in Error Measure of Association 348 <i>Applying Statistics 11.2: Measures of Association 351</i> Summary 354 / Summary of Formulas 355 / Glossary 355 / Multime Resources 356 / Problems 356 You Are the Researcher: Using SPSS to Analyze Bivariate Association with the 2009 GSS 358	dia
	pter 12 Association Between Variables Measured at the Ordinal Level	363
	Introduction 363 The Logic of Pairs 364	
	Analyzing Relationships with Gamma, Kendall's tau-b and tau-c, and Somers's <i>d</i> 366	
12.4	Gamma, Kendall's tau-b and tau-c, Somers's <i>d</i> for Bivariate Tables 373 Applying Statistics 12.1: Gamma, Kendall's tau-b, and Somers's d 380 Applying Statistics 12.2: Another Look at Computing Gamma, Kendall's tau-c, and Somers's d 381	
12.5	Interpreting Statistics: What are the Sources of Volunteerism in Canada?	382
12.6	Spearman's Rho (r_s) 384	

12.7 Testing the Null Hypothesis of "No Association" with Gamma and Spearman's Rho 388

Applying Statistics 12.3: Spearman's Rbo 389 Reading Statistics 8: Bivariate Tables and Associated Statistics 392 Summary 393 / Summary of Formulas 394 / Glossary 394 / Multimedia Resources 394 / Problems 395 You Are the Researcher: Using SPSS to Produce Ordinal-Level Measures of

You Are the Researcher: Using SPSS to Produce Ordinal-Level Measures of Association with the 2009 GSS 399

Chapter 13 Association Between Variables Measured at the Interval-Ratio Level

- 13.1 Introduction 405
- 13.2 Scattergrams 405
- 13.3 Regression and Prediction 408
- 13.4 The Computation of a and b 411
- 13.5 The Correlation Coefficient (Pearson's r) 415 Applying Statistics 13.1: Computing the Regression Coefficients and Pearson's r 417
- 13.6 Interpreting the Correlation Coefficient: r^2 418 Applying Statistics 13.2: Regression and Correlation 421
- 13.7 Testing Pearson's r for Significance 423
- 13.8 Regression with Nominal- and Ordinal-Level Variables 424
- 13.9 Interpreting Statistics: The Correlates of Crime 425 Summary 428 / Summary of Formulas 428 / Glossary 429 / Multimedia Resources 429 / Problems 430 You Are the Researcher: Using SPSS to Produce Pearson's r and the Regression Line with the 2012 CCHS 433

Part 3 Cumulative Exercises

404

438

442

Part 4 Multivariate Techniques Chapter 14 Partial Correlation and Multiple Regression and Correlation 444 14.1 Introduction 445 14.2 Partial Correlation 445 14.3 Multiple Regression: Predicting the Dependent Variable 451 14.4 Multiple Regression: Assessing the Effects of the Independent Variables 456

- 14.5 Multiple Correlation 459 Applying Statistics 14.1: Multiple Regression and Correlation 460 Applying Statistics 14.2: R² and Beta-Weights 462
- 14.6 Interpreting Statistics: Another Look at the Correlates of Crime 464 Reading Statistics 9: Multiple Regression and Partial Correlation 467
- 14.7 The Limitations of Partial Correlation and Multiple Regression and Correlation 469 Summary 470 / Summary of Formulas 471 / Glossary 471 / Multimedia Resources 472 / Problems 472 You Are the Researcher: Using SPSS for Regression Analysis with the 2012 CCHS 476

Part 4 Cumulative Exercises	
Appendix A Area Under the Normal Curve	481
Appendix B Distribution of t	485

Appendix C Distribution of Chi Square	486
Appendix D Distribution of F	487
Appendix E Using Statistics: Ideas for Research Projects	489
Appendix F An Introduction to IBM SPSS Statistics for Windows	495
Appendix G Information and Code Books for the 2012 Canadian Community Health Survey and 2009 General Social Survey	510
Answers to Odd-Numbered Computational Problems	527
Glossary	539
Index	545

Preface to the Third Canadian Edition

Sociology and the other social sciences, including political science, social work, public administration, criminology, and gerontology, are researchbased disciplines, and statistics are part of their everyday language. To join the conversation, you must learn the vocabulary of research, data analysis, and scientific thinking. Knowledge of statistics will enable you to understand the professional research literature of your discipline as well as the research reports you encounter in everyday life. Knowledge of statistics will enable you to conduct quantitative research and to contribute to the growing body of social science knowledge.

Although essential, learning statistics can be a challenge. Students in statistics courses typically have a wide range of mathematical backgrounds and an equally diverse set of career goals. They are often puzzled about the relevance of statistics for them, and, not infrequently, there is some math anxiety to deal with.

This textbook introduces statistical analysis for the social sciences while addressing these challenges. The textbook makes minimal assumptions about mathematical background (the ability to read a simple formula is sufficient preparation for virtually all of the material in the textbook), and a variety of special features help students analyze data successfully. The textbook is sufficiently flexible to be used in any program with a social science base.

The textbook is written at a level intermediate between a strictly mathematical approach and a mere "cookbook." We have not sacrificed comprehensive coverage or statistical correctness, but theoretical and mathematical explanations are kept at an elementary level, as is appropriate in a first exposure to social statistics. For example, we do not treat formal probability theory per se. Rather, the background necessary for an understanding of inferential statistics is introduced, informally and intuitively, in Chapters 4 and 5 while considering the concepts of the normal curve and the sampling distribution. The textbook makes no claim that statistics are "fun" or that the material can be mastered without considerable effort. At the same time, students are not overwhelmed with abstract proofs, formula derivations, and mathematical theory, which can needlessly frustrate the learning experience at this level.

The textbook has three primary goals, which are collectively designed to help students to develop the skills necessary to become statistically literate. The first goal is to develop an appreciation for the role of statistics in the research process. The second is to cultivate a competence to perform basic statistical calculations. The third is to develop a capability to critically read

GOALS AND FEATURES OF THE TEXTBOOK

statistical information, especially as reported in the professional research literature. The following discussion considers each goal in more detail and how it is achieved through the many special features of the textbook.

1. An Appreciation of Statistics. To appreciate statistics is to understand the relevance of statistics for research, to analyze and interpret the meaning of a statistical test, and to select and use an appropriate statistic for a given purpose and a given set of variables. This textbook develops these qualities, within the framework of an introductory course, in the following ways:

• *The relevance of statistics.* The textbook stresses the importance of statistics as a means of analyzing and manipulating data to answer research questions. This central theme of relevance is reinforced by a series of boxes labelled "Applying Statistics," each of which illustrates some specific way in which statistics can be used to answer questions.

Furthermore, the end-of-chapter problems are labelled by the social science discipline or subdiscipline from which they are drawn: \underline{SOC} for sociology, \underline{SW} for social work, \underline{PS} for political science, \underline{CJ} for criminology and criminal justice, \underline{PA} for public administration, and \underline{GER} for gerontology. By identifying problems with specific disciplines, students can more easily see the relevance of statistics to their own academic interests. (Not incidentally, they will also see that the disciplines have a large subject matter in common.)

- *Interpreting statistics*. For most students, interpretation—saying what statistics mean—is a big challenge. The ability to interpret statistics can be developed only by exposure and experience. To provide exposure, we have been careful, in the example problems, to express the meaning of the statistic in terms of the original research question. To provide experience, the end-of-chapter problems almost always call for an interpretation of the statistic calculated. To provide examples, many of the answers to odd-numbered computational problems in the back of the textbook are expressed in words as well as numbers. The "Interpreting Statistics" sections provide additional, detailed examples of how to express the meaning of statistics.
- Using statistics: Ideas for research projects. Appendix E offers ideas for independent data-analysis projects for students. The projects require students to use a computerized statistical package to analyze a data set. They can be assigned at intervals throughout the semester or at the end of the course. Each project provides an opportunity for students to practise and apply their statistical skills and, above all, to exercise their ability to understand and interpret the meaning of the statistics they produce.

2. Computational Competence. Students should emerge from their first course in statistics with the ability to perform elementary forms of data

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.

analysis—to execute a series of calculations and arrive at the correct answer. To be sure, computers and calculators have made computation less of an issue today. Yet computation and statistics are inseparable, and since social science majors frequently do not have strong quantitative backgrounds, we have included a number of features to help students cope with these challenges:

- Step-by-step computational algorithms are provided for each statistic.
- *Extensive problem sets* are provided at the end of each chapter. Most of these problems use fictitious data and are designed for ease of computation.
- *Cumulative exercises* are included at the end of each part to provide practice in choosing, computing, and analyzing statistics. These exercises present only data sets and research questions. Students must choose appropriate statistics as part of the exercise.
- *Solutions* to odd-numbered computational problems are provided so that students may check their answers.

3. Critically Read Statistical Information. A statistically literate person can comprehend and critically evaluate statistical information reported in research written by others. The development of this quality is a particular problem at the introductory level since (1) the vocabulary of professional researchers is so much more concise than the language of the textbook, and (2) the statistics featured in the literature are generally more advanced than those covered at the introductory level. To help bridge this gap, we have included a series of boxes labelled "Reading Statistics," beginning in Chapter 1. In each box, we briefly describe the reporting style typically used for the statistic in question and try to alert students about what to expect when they approach the professional literature. These inserts include excerpts from the Canadian research literature and illustrate how statistics are actually applied and interpreted by social scientists.

Additional Features. A number of other features make the textbook more meaningful for students and more useful for instructors as they help develop students' statistical literacy:

• *Readability and clarity.* The writing style is informal and accessible to students without ignoring the traditional vocabulary of statistics. Problems and examples have been written to maximize student interest and to focus on issues of concern and significance. For the more difficult material (such as hypothesis testing), students are first walked through an example problem before being confronted by formal terminology and concepts. Each chapter ends with a summary of major points and formulas and a glossary of important concepts. A list of frequently used formulas inside the front cover and a glossary of symbols inside the back cover can be used for quick reference.

- Organization and coverage. The textbook is divided into four parts. Overview sections have been included with each of the four parts of the textbook. These provide a "road map" of the material that follows and help students see why the chapters are organized as they are. Most of the coverage is devoted to the first three parts of the textbook: descriptive statistics, inferential statistics, and bivariate measures of association. The distinction between description and inference is introduced in the first chapter and maintained throughout the textbook. In selecting statistics for inclusion, we have tried to strike a balance between the essential concepts with which students must be familiar and the amount of material students can reasonably be expected to learn in their first (and perhaps only) statistics course, while bearing in mind that different instructors will naturally wish to stress different aspects of the subject. Thus, the textbook covers the full gamut of the usual statistics, with each chapter broken into subsections so that instructors may choose the particular statistics they wish to include.
- *Flow charts*. Flow charts that help students select appropriate statistics are provided in the textbook. The flow charts depict the selection process in more general terms at the beginning of each of the four parts of the textbook and in more detailed form at the beginning of each chapter.
- *Learning objectives*. Learning objectives are stated at the beginning of each chapter. These are intended to serve as "study guides" and to help students identify and focus on the most important material.
- *Review of mathematical skills.* A comprehensive review of all of the mathematical skills that will be used in this textbook (Prologue) is included. Students who are inexperienced or out of practice with mathematics may want to study this review early in the course and/or refer to it as needed. A self-test is included so that students may check their level of preparation for the course.
- *Statistical techniques and end-of-chapter problems are explicitly linked.* After a technique is introduced, students are directed to specific problems for practice and review. The "how-to-do-it" aspects of calculation are reinforced immediately and clearly.
- *End-of-chapter problems are organized progressively.* Simpler problems with small data sets are presented first. Often, explicit instructions or hints accompany the first several problems in a set. The problems gradually become more challenging and require more decision making by the student (e.g., choosing the most appropriate statistic for a certain situation). Thus, each problem set develops problem-solving abilities gradually and progressively.
- *Computer applications.* To help students take advantage of the power of the computer, this textbook integrates Version 22 of IBM SPSS Statistics for Windows, though Versions 10 through 21 are completely compatible with

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.

the textbook. IBM SPSS Statistics, colloquially referred to here as "SPSS," is a leading, state-of-the-art computerized statistical package. Alternatively, students can download a full working evaluation copy of SPSS. This free 14-day trialware can be downloaded after registering at http://www.ibm. com/software/analytics/spss/downloads.html.

The last part of each chapter, labelled "You Are the Researcher," offers demonstrations and exercises in using SPSS to produce the statistics presented in the chapter. All SPSS outputs from the demonstrations are shown in the textbook and are unedited. Students choosing to replicate the demonstration will see the same output on their computer screen. SPSS exercises are provided below the demonstrations and give students the opportunity to actually use SPSS to compute statistics. Appendix E provides additional SPSS exercises, and Appendix F gives a general introduction to SPSS.

Overall, detailed instructions and demonstrations on how to use SPSS for each exercise are provided in this textbook. SPSS can be taught at a basic level from this textbook without the aid of an SPSS supplementary text.

• *Real data.* Four databases for SPSS application give students a wide range of opportunities to practise their statistical skills on "real-life" data: 1) a shortened version of the 2009 Canadian General Social Survey (labelled *GSS.sav*); 2) a shortened version of the 2012 Canadian Community Health Survey (*CCHS.sav*); 3) the full version of the 2009 Canadian General Social Survey (*GSSFull.sav*); and 4) the full version of the 2012 Canadian Community Health Survey (*CCHSFull.sav*); and 4) the full version of the 2012 Canadian Community Health Survey (*CCHSFull.sav*). The databases are described in Appendix G and are available in SPSS format (i.e., the ".sav" file format) on the website for the textbook.

KEY CHANGES IN THE THIRD CANADIAN EDITION

This edition of the textbook remains focused on developing the skills necessary for students to become statistically literate. It is written for students with varying levels of mathematical proficiency, providing an accessible yet comprehensive examination of statistics, and continues to expose students to statistics from a uniquely Canadian perspective. With this in mind, various changes were made in the third edition. The following are the most important changes:

- The textbook has been redesigned to enhance readability and ease of use.
- A new feature entitled "One Step at a Time" has been added. This feature provides a step-by-step guide for solving each formula in the textbook.
- Titles have been added to the "Applying Statistics" feature (formerly called "Application" boxes) to call attention to the topic being highlighted.
- The "Reading Statistics" inserts have been updated. Likewise, data cited in the text have been updated.

- Supplemental chapters on "dummy" variable and logistic regression analysis are now available at http://www.nelson.com/statistics3ce.
- The textbook, at the time of printing, uses the latest version of IBM SPSS Statistics for Windows (Version 22), along with the new 2012 Canadian Community Health Survey data set.
- A manual for Stata, a statistical software package, is now available for the textbook. Like SPSS, Stata provides a variety of tools for statistical analyses that are easy to learn and use. The manual, written by Jenny Godley and her associates at the University of Calgary, can be downloaded for free from the website for the textbook. The manual is accompanied by two databases, allowing students to practise their statistical skills in a Stata computing environment.

Founded in 2000 by economist and Stanford professor Paul Romer, **Aplia**TM is an educational technology company dedicated to improving learning by increasing student effort and engagement. Currently, **Aplia** products have been used by more than a million students at over 1300 institutions. **Aplia** offers a way for you to stay on top of your coursework with regularly scheduled homework assignments that increase your time on task and give you prompt feedback. Interactive tools and additional content are provided to further increase your engagement and understanding. The **Aplia** course for *Statistics*, 3Ce was prepared by text author Steven Prus. See www.aplia.com for more information.



The **Student Companion Site** for *Statistics* at http://www.nelson.com/ statistics3ce is a comprehensive, resource-rich location for students to find pertinent information. As well as SPSS data sets used for the end-of-chapter SPSS demonstrations and exercises in the textbook, they'll find additional review and study materials, including quizzes, supplemental chapters, tables of frequently used formulas and random numbers, and more!

ABOUT THE NELSON EDUCATION TEACHING ADVANTAGE (NETA)



The **Nelson Education Teaching Advantage (NETA)** program delivers research-based instructor resources that promote student engagement and higher-order thinking to enable the success of Canadian students and educators. To ensure the high quality of these materials, all Nelson ancillaries have been professionally copy edited.

Be sure to visit Nelson Education's **Inspired Instruction** website at http://www.nelson.com/inspired/ to find out more about NETA. Don't miss the testimonials of instructors who have used NETA supplements and seen student engagement increase!

Assessing Your Students: *NETA Assessment* relates to testing materials. **NETA Test Bank** authors create multiple-choice questions that reflect

STUDENT RESOURCES

10

Engage. Prepare. Educate.

NEL

research-based best practices for constructing effective questions and testing not just recall but also higher-order thinking. Our guidelines were developed by David DiBattista, psychology professor at Brock University and 3M National Teaching Fellow, whose research has focused on multiple-choice testing. All Test Bank authors receive training at workshops conducted by Prof. DiBattista, as do the copy editors assigned to each Test Bank. A copy of *Multiple Choice Tests: Getting Beyond Remembering*, Prof. DiBattista's guide to writing effective tests, is included with every Nelson Test Bank.

Teaching Your Students: *NETA Presentation* has been developed to help instructors make the best use of Microsoft[®] PowerPoint[®] in their classrooms. With a clean and uncluttered design developed by Maureen Stone of Stone-Soup Consulting, **NETA PowerPoints** feature slides with improved readability, more multi-media and graphic materials, activities to use in class, and tips for instructors on the Notes page. A copy of *NETA Guidelines for Classroom Presentations* by Maureen Stone is included with each set of PowerPoint slides.

INSTRUCTOR RESOURCES



All NETA and other key instructor ancillaries are provided on the *Instructor Companion Site* at http://www.nelson.com/statistics3ce, giving instructors the ultimate tool for customizing lectures and presentations.

NETA Test Bank: This resource was written by Markus Schafer, University of Toronto. It includes over 900 multiple-choice questions written according to NETA guidelines for effective construction and development of higher-order questions. The Test Bank was copy edited by a NETA-trained editor. Also included are over 30 problems.

The NETA Test Bank is available in a new, cloud-based platform. **Testing Powered by Cognero**[®] is a secure online testing system that allows you to author, edit, and manage test bank content from any place you have Internet access. No special installations or downloads are needed, and the desktopinspired interface, with its drop-down menus and familiar, intuitive tools, allows you to create and manage tests with ease. You can create multiple test versions in an instant and import or export content into other systems. Tests can be delivered from your learning management system, your classroom, or wherever you want.

NETA PowerPoint: Microsoft[®] PowerPoint[®] lecture slides for every chapter have been created by Steven Prus, Carleton University. There is an average of 25 slides per chapter, many featuring key figures, tables, and photographs from *Statistics*, 3Ce. NETA principles of clear design and engaging content have been incorporated throughout, making it simple for instructors to customize the deck for their courses.

Image Library: This resource consists of digital copies of figures, short tables, and photographs used in the book. Instructors may use these jpegs to customize the NETA PowerPoint or create their own PowerPoint presentations.

Instructor's Manual: The Instructor's Manual to accompany *Statistics*, 3Ce has been prepared by Parvinder Hira-Friesen, University of Calgary. This manual contains answers to even-numbered end-of-chapter problems, detailed answers to selected textbook problems, and answers to even-numbered textbook cumulative exercises. It also provides a new "What can I do in class?" feature that includes 2–3 lesson ideas per chapter, and a "What can I do online?" feature, which includes 2–3 suggestions per chapter for engaging and instructing students using Web resources.

DayOne: Day One—Prof InClass is a PowerPoint presentation that instructors can customize to orient students to the class and their text at the beginning of the course.

SPSS Datasets: The SPSS Datasets for Statistics can be found at http:// www.nelson.com/statistics3ce. These data sets were written for use with Version 22 of IBM SPSS Statistics for Windows, but Versions 10 through 21 are completely compatible.

Supplemental Chapters: Several supplemental chapters to accompany the text can be found at http://www.nelson.com/statistics3ce.

ACKNOWLEDGMENTS FOR THE THIRD CANADIAN EDITION

I would like to first and foremost acknowledge and thank Dr. Teresa Abada, Western University; Dr. Alex Bierman, University of Calgary; Dr. Jenny Godley, University of Calgary; Dr. Stephen Gyimah, Queen's University; Dr. Michelle Maroto, University of Alberta; Dr. Owen Temby, McGill University; and Dr. Michael Weinrath, University of Winnipeg for their very helpful comments and advice in writing this textbook. The textbook and its various supplements and ancillaries have also greatly benefited from the reviews, suggestions, and guidance provided by many others.

Along with others who prefer to remain anonymous, they include:

Nahum Arguera, University of Calgary Silvia Bartolic, University of British Columbia Tuna Baskoy, Ryerson University Gerald Bierling, McMaster University Aaron Brauer, Concordia University Jonah Butovsky, Brock University James Frideres, University of Calgary Jenny Godley, University of Calgary David A. Hay, University of Calgary David A. Hay, University of Saskatchewan Robert Hiscott, University of Waterloo Carl Keane, Queen's University Sean Lauer, University of British Columbia Nikolaos Liodakis, Wilfrid Laurier University William Marshall, Western University Jason Novick, University of Calgary Brenda O'Neill, University of Calgary Anton Oleinik, Memorial University of Newfoundland Andrew Patterson, University of British Columbia Andrea Perrella, Wilfrid Laurier University Gail Perry, Western University Olivia Skidmore, University of Calgary Ann Marie Sorenson, University of Toronto Thomas Varghese, University of Alberta Weiguo Zhang, University of Toronto

Maya Castle, Jessica Freedman, and Susan Calvert, at Nelson Education, provided the utmost effort and dedication to this project, and I am very thankful for their support. Finally, I would like to thank my three wonderful children, Laura, Mandy, and Michael. I dedicate this textbook to them.

We lastly acknowledge and are grateful to the Literary Executor of the late Sir Ronald A. Fisher, F.R.S., to Dr. Frank Yates, F.R.S., and to Longman Group Ltd., London, for permission to reprint Appendixes B, C, and D from their book *Statistical Tables for Biological, Agricultural and Medical Research* (6th ed., 1974). We are also grateful to Statistics Canada, for providing the General Social Survey and Canadian Community Health Survey data sets used in this textbook.

Steven G. Prus

Prologue: Basic Mathematics Review

You will probably be relieved to hear that first courses in statistics are not particularly mathematical and do not stress computation per se. While you will encounter many numbers to work with and numerous formulas to use, the major emphasis will be on understanding the role of statistics in research and the logic by which we attempt to answer research questions empirically. You will also find that, at least in this textbook, the example problems and many of the homework problems have been intentionally simplified so that the computations will not unduly distract you from the task of understanding the statistics themselves.

On the other hand, you may regret to learn that there is, inevitably, some arithmetic that you simply cannot avoid if you want to master this material. It is likely that some of you haven't had any math in a long time, others have convinced themselves that they just cannot do math under any circumstances, and still others are just rusty and out of practice. All of you will find that even the most complex and intimidating operations and formulas can be broken down into simple steps. If you have forgotten how to cope with some of these steps or are unfamiliar with these operations, this section is designed to ease you into the skills you will need to do all

of the computation in this textbook.

A calculator is a virtual necessity for this textbook. While you could do all the arithmetic by hand, the calculator will save you time and effort and is definitely worth the small investment. Incidentally, you do not need to invest in any of the sophisticated, more expensive models you might see for sale. A square root function is the only extra feature you really need. Of course, if you want the additional features, such as memories and preprogrammed functions, by all means spend the extra money and learn how to use them. However, a simple, inexpensive calculator will work fine for all of the problems in this textbook.

Along the same lines, many of you probably have access to computers and statistical packages. If so, take the time now to learn how to use them, because they will eventually save you time and effort. This textbook includes a guide to a statistical package called IBM SPSS Statistics, but many other programs are available that will help you accomplish the goals of saving time and avoiding drudgery while generating precise and accurate results.

In summary, you should find a way at the beginning of this coursewith a calculator, a statistical package, or both-to minimize the tedium and hassle of mere computing. This will permit you to devote maximum effort

CALCULATORS AND COMPUTERS

NEL

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it. to the truly important goal of increasing your understanding of the meaning of statistics in particular and social research in general.

Statistics are a set of techniques by which we can describe, analyze, and manipulate variables. A variable is a trait that can change values from case to case or from time to time. Examples of variables include height, weight, level of education, and political party preference. The possible values associated with a given variable might be numerous (e.g., income) or relatively few (e.g., gender). We will often use symbols, usually the letter *X*, to refer to variables in general or to a specific variable.

Sometimes we will need to refer to a specific value or set of values of a variable. This is usually done with the aid of subscripts. So, the symbol X_1 (read "X-sub-one") would refer to the first score in a set of scores, X_2 ("X-sub-two") to the second score, and so forth. Also, we will use the subscript *i* to refer to all the scores in a set. Thus, the symbol X_i (read "X-sub-eye") refers to all of the scores associated with a given variable (e.g., the test grades of a particular class).

OPERATIONS

VARIABLES AND SYMBOLS

You are all familiar with the four basic mathematical operations of addition, subtraction, multiplication, and division and the standard symbols $(+, -, \times, \div)$ used to denote them. Some of you may not be aware, however, that the latter two operations can be symbolized in a variety of ways. For example, the operation of multiplying some number *a* by some number *b* may be symbolized in (at least) six different ways:

a	X	b
а	•	b
а	*	b
C	ıb	
a(b)
(a))(i	b)

In this textbook, we will commonly use the "adjacent symbols" format (i.e., ab), the conventional times sign (×), or adjacent parentheses to indicate multiplication. On most calculators and computers, the asterisk (*) is the symbol for multiplication.

The operation of division can also be expressed in several different ways. In this textbook, we will use either of these two methods:

$$a/b$$
 or $\frac{a}{b}$

Several of the formulas with which we will be working require us to find the square of a number. To do this, simply multiply the number by itself. This operation is symbolized as X^2 (read "*X* squared"), which is the same thing as (*X*)(*X*). If *X* has a value of 4, then

$$X^2 = (X)(X) = (4)(4) = 16$$

or we could say that "4 squared is 16."

The square root of a number is the value that, when multiplied by itself, results in the original number. So the square root of 16 is 4 because (4)(4) is 16. The operation of finding the square root of a number is symbolized as

 \sqrt{X}

Be sure you have access to a calculator with a built-in square root function.

A final operation with which you should be familiar is summation, or the addition of the scores associated with a particular variable. When a formula requires the addition of a series of scores, this operation is usually symbolized as ΣX_i . The symbol Σ is the uppercase Greek letter sigma and stands for "the summation of." So the combination of symbols ΣX_i means "the summation of all the scores" and directs us to add the value of all the scores for that variable. If four people had family sizes of 2, 4, 5, and 7, then the summation of these four scores for this variable could be symbolized as

$$\sum X_i = 2 + 4 + 5 + 7 = 18$$

The symbol Σ is an operator, just like the + or × signs. It directs us to add all of the scores on the variable indicated by the *X* symbol.

There are two other common uses of the summation sign, and unfortunately, the symbols denoting these uses are not, at first glance, sharply different from each other or from the symbol used above. A little practice and some careful attention to these various meanings should minimize the confusion. The first set of symbols is $\sum X_i^2$, which means "the sum of the squared scores." This quantity is found by *first* squaring each of the scores and *then* adding the squared scores together. A second common set of symbols will be $(\sum X_i)^2$, which means "the sum of the scores, squared." This quantity is found by *first* summing the scores and *then* squaring the total.

These distinctions might be confusing at first, so let's see if an example helps clarify the situation. Suppose we had a set of three scores: 10, 12, and 13. So,

$$X_i = 10, 12, 13$$

The sum of these scores would be indicated as

$$\sum X_i = 10 + 12 + 13 = 35$$

The sum of the squared scores would be

$$\sum X_i^2 = (10)^2 + (12)^2 + (13)^2 = 100 + 144 + 169 = 413$$

Take careful note of the order of operations here. First, the scores are squared one at a time and then the squared scores are added. This is a completely different operation from squaring the sum of the scores:

$$(\sum X_{i})^{2} = (10 + 12 + 13)^{2} = (35)^{2} = 1,225$$

To find this quantity, first the scores are summed and then the total of all the scores is squared. The value of the sum of the scores squared (1,225) is not the same as the value of the sum of the squared scores (413). In summary, the operations associated with each set of symbols are:

Symbols	Operations
ΣX_i	Add the scores
ΣX_i^2	First square the scores and then add the squared scores
$(\Sigma X_i)^2$	First add the scores and then square the total

OPERATIONS WITH NEGATIVE NUMBERS

A number can be either positive (if it is preceded by a + sign or by no sign at all) or negative (if it is preceded by a - sign). Positive numbers are greater than zero, and negative numbers are less than zero. It is very important to keep track of signs because they will affect the outcome of virtually every mathematical operation. This section will briefly summarize the relevant rules for dealing with negative numbers. First, adding a negative number is the same as subtraction. For example,

$$3 + (-1) = 3 - 1 = 2$$

Second, subtraction changes the sign of a negative number:

$$3 - (-1) = 3 + 1 = 4$$

Note the importance of keeping track of signs here. If you neglected to change the sign of the negative number in the second expression, you would arrive at the wrong answer.

For multiplication and division, there are various combinations of negative and positive numbers you should be aware of. For purposes of this textbook, you will rarely have to multiply or divide more than two numbers at a time, and we will confine our attention to this situation. Ignoring the case of all positive numbers, this leaves several possible combinations. A negative number times a positive number results in a negative value:

$$(-3)(4) = -12$$

or

$$(3)(-4) = -12$$

A negative number times a negative number is always positive:

(-3)(-4) = 12

NEL

Division follows the same patterns. If there is a single negative number in the calculations, the answer will be negative. If both numbers are negative, the answer will be positive. So,

 $\frac{-4}{2} = -2$

and

but

$$\frac{4}{-2} = -2$$

 $\frac{-4}{-2} = 2$

Note that negative numbers do not have square roots, because multiplying a number by itself cannot result in a negative value. Squaring a negative number always results in a positive value (see the multiplication rules above).

A possible source of confusion in computation involves the issues of accuracy and rounding off. People work at different levels of accuracy and precision and, for this reason alone, may arrive at different answers to problems. This is important, because if you work at one level of precision and the textbook (or your instructor or your study partner) works at another, the resulting solutions will be at least slightly different. You may sometimes think you've gotten the wrong answer when all you've really done is round off at a different place in the calculations or in a different way.

There are two issues here: *when* to round off and *how* to round off. In this textbook, we have followed the convention of working with as much accuracy as our calculator or statistics package will allow and then rounding off to two places of accuracy (two places beyond the decimal point) at the very end. If a set of calculations is lengthy and requires the reporting of intermediate sums or subtotals, we will round the subtotals off to two places also.

In terms of how to round off, begin by looking at the digit immediately to the right of the last digit you want to retain. If you want to round off to 100ths (two places beyond the decimal point), look at the digit in the 1000ths place (three places beyond the decimal point). If that digit is greater than 5, round up. For example, 23.346 would round off to 23.35. If the digit to the right is less than 5, round down. So, 23.343 would become 23.34. If the digit to the right is 5, round up if the digit immediately to the left is even and round down if the digit is odd. So, 23.345 would become 23.35 and 23.355 would round to 23.35.

ACCURACY AND ROUNDING OFF

Let's look at some more examples of how to follow the rounding rules stated above. If you are calculating the mean value of a set of test scores and your calculator shows a final value of 83.459067, and you want to round off to two places beyond the decimal point, look at the digit three places beyond the decimal point. In this case the value is 9 (greater than 5), so we would round the second digit beyond the decimal point up and report the mean as 83.46. If the value had been 83.453067, we would have reported our final answer as 83.45. A value of 83.455067 would round to 83.45067 would be 83.45.

A mathematical formula is a set of directions, stated in general symbols, for calculating a particular statistic. To "solve a formula" means that you must replace the symbols with the proper values and then manipulate the values through a series of calculations. Even the most complex formula can be rendered manageable if it is broken down into smaller steps. Working through these steps requires some knowledge of general procedure and the rules of precedence of mathematical operations. This is because the order in which you perform calculations may affect your final answer. Consider the following expression:

2 + 3(4)

Note that if you do the addition first, you will evaluate the expression as

$$5(4) = 20$$

but if you do the multiplication first, the expression becomes

$$2 + 12 = 14$$

Obviously, it is crucial to complete the steps of a calculation in the correct order.

The basic rules of precedence are to find all squares and square roots first, then do all multiplication and division, and finally complete all addition and subtraction. So the following expression:

$$8 + 2 \times 2^2/2$$

would be evaluated as

$$8 + 2 \times \frac{4}{2} = 8 + \frac{8}{2} = 8 + 4 = 12$$

The rules of precedence may be overridden when an expression contains parentheses. Solve all expressions within parentheses before applying the rules stated above. For most of the complex formulas in this textbook,

FORMULAS, COMPLEX OPERATIONS, AND THE ORDER OF OPERATIONS

the order of calculations will be controlled by the parentheses. Consider the following expression:

$$(8+2) - \frac{4(3)^2}{(8-6)}$$

Resolving the parenthetical expressions first, we would have

$$(10) - 4 \times 9/(2) = 10 - 36/2 = 10 - 18 = -8$$

Without the parentheses, the same expression would be evaluated as

$$8 + 2 - 4 \times 3^{2}/8 - 6$$

= 8 + 2 - 4 × 9/8 - 6
= 8 + 2 - 36/8 - 6
= 8 + 2 - 4.5 - 6
= 10 - 10.5
= -0.5

A final operation you will encounter in some formulas in this textbook involves denominators of fractions that themselves contain fractions. In this situation, solve the fraction in the denominator first and then complete the division. For example,

would become

$$\frac{15-9}{6/2} = \frac{6}{3} = 2$$

 $\frac{15-9}{6/2}$

When you are confronted with complex expressions such as these, don't be intimidated. If you're patient with yourself and work through them step by step, beginning with the parenthetical expression, even the most imposing formulas can be managed.

EXERCISES

You can use the problems below as a "self-test" on the material presented in this review. If you can handle these problems, you're ready to do all of the arithmetic in this textbook. If you have difficulty with any of these problems, please review the appropriate section of this prologue. You might also want to use this section as an opportunity to become more familiar with your calculator. Answers are given on the next page, along with some commentary and some reminders.

1. Complete each of the following:

a. 17 × 3 = **b.** 17(3) = **c.** (17)(3) =

Copyright 2016 Nelson Education Ltd. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Nelson Education reserves the right to remove additional content at any time if subsequent rights restrictions require it.

- **d.** 17/3 =**e.** $(42)^2 =$ **f.** $\sqrt{113} =$
- **2.** For the set of scores (X_i) of 50, 55, 60, 65, and 70, evaluate each of the expressions below:

 $\sum X_i =$ $\sum X_i^2 =$ $(\sum X_i)^2 =$

- **3.** Complete each of the following:
 - **a.** 17 + (-3) + (4) + (-2) =
 - **b.** 15 3 (-5) + 2 =
 - **c.** (-27)(54) =
 - **d.** (113)(-2) =
 - **e.** (-14)(-100) =
 - **f.** -34/-2 =
 - **g.** 322/-11 =
- 4. Round off each of the following to two places beyond the decimal point:
 - **a.** 17.17532
 - **b.** 43.119
 - **c.** 1,076.77337
 - d. 32.4651152301
 - e. 32.4751152301
- **5.** Evaluate each of the following:
 - **a.** (3 + 7)/10 = **b.** 3 + 7/10 =**c.** $\frac{(4-3) + (7+2)(3)}{(4+5)(10)} =$

ANSWERS TO EXERCISES

1. a. 51 **b.** 51 **c.** 51

(The obvious purpose of these first three problems is to remind you that there are several different ways of expressing multiplication.) **d.** 5.67 (Note the rounding off.) **e.** 1,764 **f.** 10.63

2. The first expression translates to "the sum of the scores," so this operation would be

$$\Sigma X_i = 50 + 55 + 60 + 65 + 70 = 300$$

The second expression is the "sum of the squared scores." So

$$\sum X_i^2 = (50)^2 + (55)^2 + (60)^2 + (65)^2 + (70)^2$$

$$\sum X_i^2 = 2,500 + 3,025 + 3,600 + 4,225 + 4,900$$

$$\sum X_i^2 = 18,250$$

The third expression is "the sum of the scores, squared":

$$(\sum X_i)^2 = (50 + 55 + 60 + 65 + 70)^2$$

 $(\sum X_i)^2 = (300)^2$
 $(\sum X_i)^2 = 90,000$

Remember that $\sum X_i^2$ and $(\sum X_i)^2$ are two completely different expressions with very different values.

- 3. a. 16 b. 19 (Remember to change the sign of -5.) c. -1,458 d. -226 e. 1,400 f. 17 g. -29.27
 4. a. 17.17 b. 43.12 c. 1,076.77 d. 32.47 e. 32.47
- **5. a.** 1 **b.** 3.7 (Note again the importance of parentheses.) **c.** 0.31